Tensore di Finger
e modello neo-Hookiano

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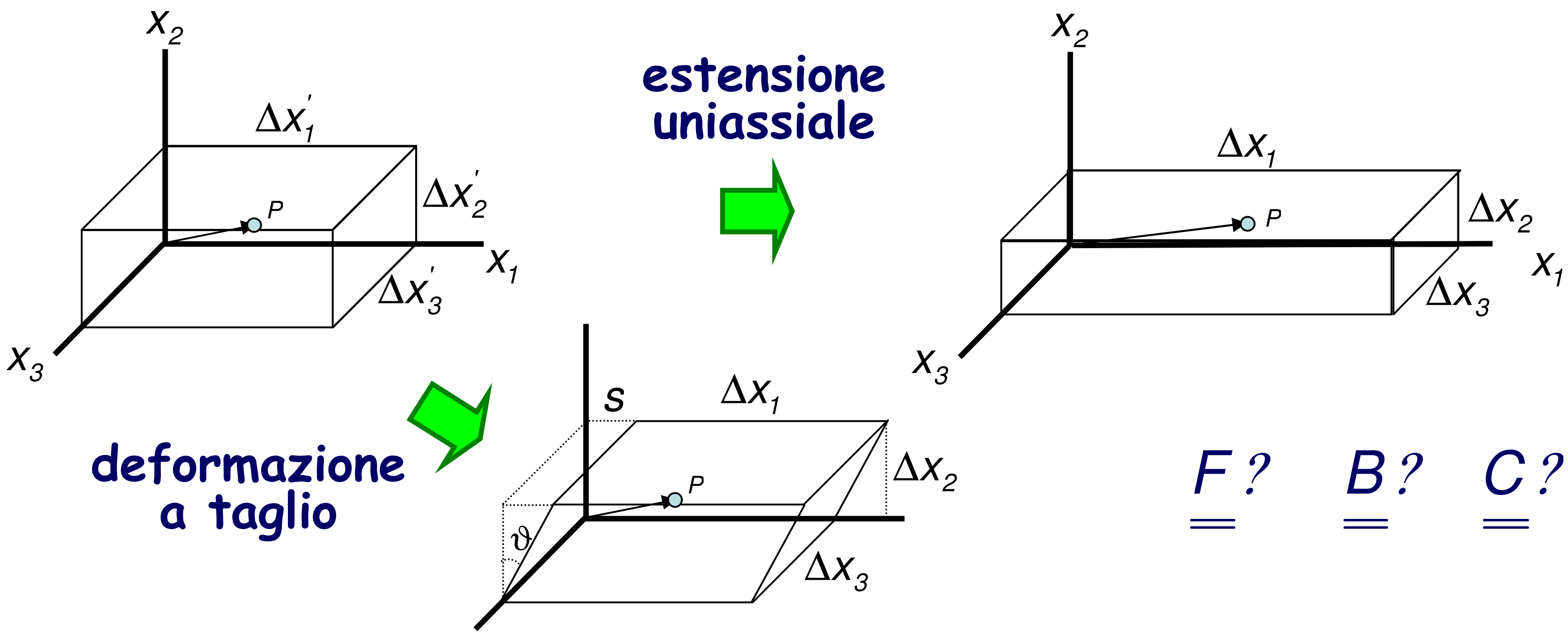
Tensore di Finger

Il tensore di Finger è una misura del cambiamento locale di area che accompagna la deformazione

$$\underline{\underline{B}} = \underline{\underline{F}}^T \bullet \underline{\underline{F}} = \underline{\underline{C}}^{-1} \qquad B_{ij} = \sum_k F_{ik} F_{jk} = \sum_k \frac{\partial r_i}{\partial x_k} \frac{\partial r_j}{\partial x_k}$$

analisi di deformazioni estensionali e di taglio



estensione uniassiale

$$\Delta x_1 = \alpha_1 \Delta x_1' \qquad \Delta x_2 = \alpha_2 \Delta x_2' \qquad \Delta x_3 = \alpha_3 \Delta x_3'$$

$$\alpha_2 = \alpha_3$$

$$\Delta x_1 \Delta x_2 \Delta x_3 = \Delta x_1' \Delta x_2' \Delta x_3'$$

$$\alpha_1 \alpha_2 \alpha_3 = 1$$

$$\alpha_1 \alpha_2^{\;2} = 1 \qquad \alpha_2 = \frac{1}{\sqrt{\alpha_1}} = \alpha_3$$

$$\underline{\underline{F}} = \begin{vmatrix} \alpha_1 & 0 & 0 \\ 0 & \alpha_2 & 0 \\ 0 & 0 & \alpha_3 \end{vmatrix} \qquad \underline{\underline{F}} = \begin{vmatrix} \alpha_1 & 0 & 0 \\ 0 & 1/\sqrt{\alpha_1} & 0 \\ 0 & 0 & 1/\sqrt{\alpha_1} \end{vmatrix} = \underline{\underline{F}}^T$$

deformazione a taglio

$$\underline{\underline{F}} = \begin{vmatrix} 1 & \gamma & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} \qquad \gamma = \frac{s}{\Delta x_2} = tg\,\delta$$

estensione uniassiale

$$\underline{\underline{B}} = \underline{\underline{C}} = \underline{\underline{F}} \bullet \underline{\underline{F}} = \underline{\underline{F}}^2 = \begin{vmatrix} \alpha_l^2 & 0 & 0 \\ 0 & 1/\alpha_l & 0 \\ 0 & 0 & 1/\alpha_l \end{vmatrix}$$

deformazione a taglio

$$\underline{\underline{B}} = \begin{vmatrix} 1 & \gamma & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} \bullet \begin{vmatrix} 1 & 0 & 0 \\ \gamma & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = \begin{vmatrix} 1+\gamma^2 & \gamma & 0 \\ \gamma & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

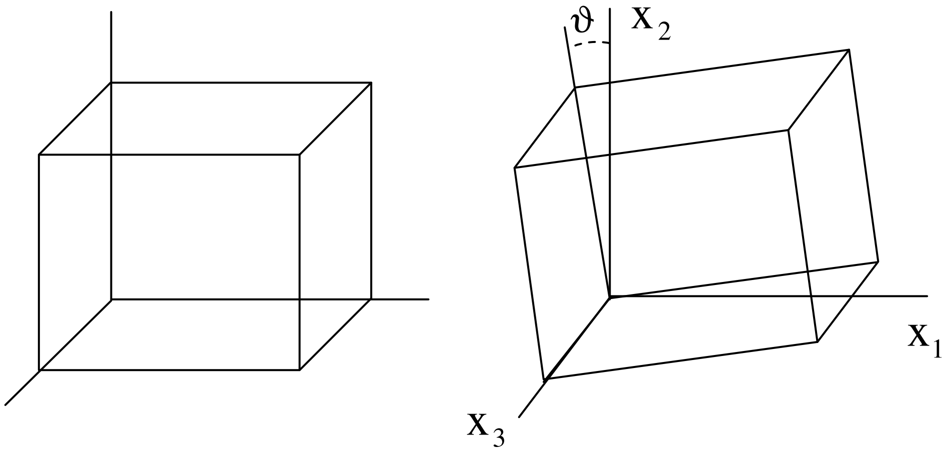
$$\underline{\underline{C}} = \begin{vmatrix} 1 & \gamma & 0 \\ \gamma & 1+\gamma^2 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

strain tensor

$$\underline{\underline{E}} = \underline{\underline{B}} - \underline{\underline{\delta}}$$

$$\underline{\underline{E}} = \begin{vmatrix} \alpha_l^2 - 1 & 0 & 0 \\ 0 & 1/\alpha_l - 1 & 0 \\ 0 & 0 & 1/\alpha_l - 1 \end{vmatrix} \qquad \underline{\underline{E}} = \begin{vmatrix} \gamma^2 & \gamma & 0 \\ \gamma & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix}$$

rotazione rigida



$$x_1 = x_1' cos\vartheta - x_2' sin\vartheta$$

$$x_1 = x_1' sin\vartheta + x_2' cos\vartheta$$

$$x_3 = x_3'$$

$$\underline{\underline{F}} = \begin{vmatrix} cos\vartheta & -sin\vartheta & 0 \\ sin\vartheta & cos\vartheta & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$B_{11} = \sum_k F_{1k} F_{1k} = cos^2\vartheta + sin^2\vartheta = 1$$

$$B_{12} = \sum_k F_{1k} F_{2k} = sin\vartheta cos\vartheta - sin\vartheta cos\vartheta = 0$$

$$\underline{\underline{B}} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} \qquad \underline{\underline{C}} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} \qquad \underline{\underline{E}} = \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix}$$

Neo-Hookean model

$$\underline{\underline{\sigma}} = -p\underline{\underline{\delta}} + G\underline{\underline{B}}$$

G modulo elastico di taglio

estensione uniassiale

$$\sigma_{11} = -p + G\alpha_1^{\,2}$$

$$\sigma_{22} = \sigma_{33} = -p + \frac{G}{\alpha_1}$$

$$\sigma_{11} = \frac{f_1}{a_1} = \frac{f_1}{\Delta x_2 \Delta x_3}$$

f_1 forza di trazione

a_1 sezione del campione deformato

in assenza di forze di trazione sulle facce laterali:

$$\sigma_{22} = \sigma_{33} = 0 \;\Rightarrow\; p = \frac{G}{\alpha_1} \;\Rightarrow\; \sigma_{11} = G\left(\alpha_1^{\,2} - \frac{1}{\alpha_1}\right)$$

$$\Downarrow$$

riferendo f_1 alla sezione originaria:

$$V = a_1 \Delta x_1 = a_1' \Delta x_1' \;\Rightarrow\; a_1 = \frac{a_1'}{\alpha_1} \;\Rightarrow\; \sigma_{11} = \frac{f_1}{a_1'} = G\left(\alpha_1 - \frac{1}{\alpha_1^{\,2}}\right)$$

$$\Downarrow$$

$$\alpha_1 = \frac{\Delta x_1}{\Delta x_1'} = \frac{L}{L'} \;\Rightarrow\; \varepsilon = \frac{L - L'}{L'} = \alpha_1 - 1 \;\Rightarrow\; \sigma_{11} = G\frac{3\varepsilon + 3\varepsilon^2 + \varepsilon^3}{1 + \varepsilon}$$

$\varepsilon \to 0$ $\Downarrow$ piccole deformazioni

$$\sigma_{11} = 3G\varepsilon = E\varepsilon$$

per materiali isotropi comprimibili

$$E = 2G(\nu + 1)$$

ν **rapporto di Poisson (0 ÷ 0.5)**

deformazione di taglio

$$\sigma_{11} = -p + G(1 + \gamma^2)$$
$$\sigma_{22} = \sigma_{33} = -p + G$$
$$\sigma_{12} = \sigma_{21} = G\gamma$$

$$\sigma_{11} - \sigma_{22} = G\gamma^2$$
$$\sigma_{22} - \sigma_{33} = 0$$

esempi di applicazione a gomme